NOTIZEN

Stationary States of the Classically Radiating Electron in an Attractive Potential

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It is shown by explicit numerical calculations, that the recently proposed non-local equations of motion, which can be supplied by a modified form of the well-known Caldirola equation, all admit the possibility of stationary radiationless motions in an attractive potential.

In a recent paper [1], the following equation of motion for the classically radiating electron was proposed:

$$m_{\text{mech}}\, c^2\, \dot{u}^\lambda{}_{(s)} + m_{\text{el}}\, c^2 \{\dot{u}^\lambda{}_{(s-\Delta s)} - (\hat{u}\, u)\, u^\lambda{}_{(s)}\} = K^\lambda{}_{(s)}\,, \tag{1}$$

where the world line $x^\lambda = z^\lambda{}_{(s)}$ of the particle has been parametrized with proper time s ($\mathrm{d}s^2 = \mathrm{d}x^\nu\, \mathrm{d}x_\nu$) and $\hat{u}^\lambda{}_{(s)} \equiv \dot{u}^\lambda{}_{(s-\Delta s)}$ is the four-acceleration shifted backward in proper time by the constant amount Δs. This equation has been studied extensively [2] in its one-dimensional form (linear motion: $\{u^\lambda\} = \{\text{Cosh}\, w_{(s)}; 0, 0, \text{Sinh}\, w_{(s)}\}$)

$$m_{\text{mech}}\, c^2\, \dot{w}_{(s)} + m_{\text{el}}\, c^2\, \dot{w}_{(s-\Delta s)} \cdot \text{Cosh}\,[w_{(s)} - w_{(s-\Delta s)}] = K_{(s)}\,, \tag{2}$$

and it could be shown that, under suitable initial conditions, the electron performs damped self-oscillations, if an external force K has ceased to act upon it.

The purely electromagnetic version

$$m_{\text{el}}\, c^2 \{\hat{\dot{u}}^\lambda - (\hat{u}\, u)\, u^\lambda\} = K^\lambda \tag{3}$$

was studied in two earlier papers [3, 4], and we want to add here a third form, namely

$$m_{\text{mech}}\, c^2\, \dot{u}^\lambda{}_{(s)} + m_{\text{el}}\, c^2 \left\{\frac{\Delta u^\lambda}{\Delta s} - \left(u \cdot \frac{\Delta u}{\Delta s}\right) u^\lambda{}_{(s)}\right\} = K^\lambda{}_{(s)} \tag{4}$$

with $m_{\text{el}}\, c^2\, \Delta s = \frac{4}{3} Z^2$ and $\Delta u^\lambda := u^\lambda{}_{(s)} - u^\lambda{}_{(s-\Delta s)}$, the parameter Δs being held fixed again. If we put $m_{\text{mech}} = 0$ in (4), we have a pure finite-differences equation, which was proposed by Caldirola [5] some years ago. It is, however, essential here that the differential term $m_{\text{mech}}\, c^2\, \dot{u}^\lambda{}_{(s)}$ enters the equation; otherwise the pure differences equation would not determine the particle trajectory uniquely in spatially varying force fields. This deficiency of the finite-differences form was apologized by Caldirola through some indeterminacy considerations and the ad hoc introduction of a "transmission law". We prefer here the introduction of a differential term to make the solutions unique.

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Now, as we have pointed out earlier [4], there exists the possibility of stationary, radiationless motions in equations of the kind (1) to (4). These must be periodic motions, because with

$$u^\lambda{}_{(s)} = u^\lambda{}_{(s-\Delta s)} \Rightarrow \dot{u}^\lambda{}_{(s)} = \dot{u}^\lambda{}_{(s-\Delta s)} \tag{5}$$

all the above equations reduce to the non-radiating limit

$$m\, c^2\, \dot{u}^\lambda{}_{(s)} = K^\lambda{}_{(s)}\,, \quad \text{resp.} \quad m\, c^2\, \dot{w}_{(s)} = K_{(s)}\,. \tag{6}$$

But, since for sufficiently smooth trajectories all our equations can be approximated by the Lorentz-Dirac equation

$$m\, c^2\, \dot{u}^\lambda{}_{(s)} = K^\lambda{}_{(s)} + \tfrac{2}{3} Z^2 \{\ddot{u}^\lambda{}_{(s)} + (\dot{u}\, \dot{u})\, u^\lambda{}_{(s)}\}\,, \tag{7}$$

and this equation is capable of accounting for the energy-momentum loss due to radiation, there must occur the following qualitative phenomenon: If a charged particle, described by one of the Eqs. (1), (3), or (4), approaches an attractive center, it looses energy and momentum by virtue of the emission of radiation, but if the radiating particle is close enough to the center, the motion becomes more and more radiationless and ultimately goes over in a completely radiationless, periodic trajectory around the attractive center.

Clearly, it is exactly this picture, which one has in mind when one thinks of an electron falling down to the lowest Bohr's orbit in a hydrogen atom under emission of radiation; but it is usually argued that only quantum mechanics is able to explain, why the electron does not plunge directly into the proton but is held (on an average) apart from it in the distance of Bohr's radius.

In contradiction to this generally accepted point of view, we have found radiationless periodic motions for our classical equations in the attractive potential Φ

$$\Phi_{(r)} = \frac{Z^*}{\sqrt{r_0{}^2 + r^2}} \Rightarrow K_{(r)} = -Z\, \frac{\partial \Phi}{\partial r} = \frac{Z\, Z^*\, r}{\sqrt{r_0{}^2 + r^2}^{\,3}}\,, \tag{8}$$

where we have restricted ourselves to one-dimensional motion (in radial direction). We have investigated in this respect all equations mentioned above, but for the sake of brevity we will present here those results refferring to the modified Caldirola equation (4). That equation can be written for one-dimensional motion as

$$m_{\text{mech}}\, c^2\, \dot{w}_{(s)} + \frac{m_{\text{el}}\, c^2}{\Delta s} \operatorname{Sinh}\left[w_{(s)} - w_{(s-\Delta s)}\right] = K_{(s)}\,, \tag{9}$$

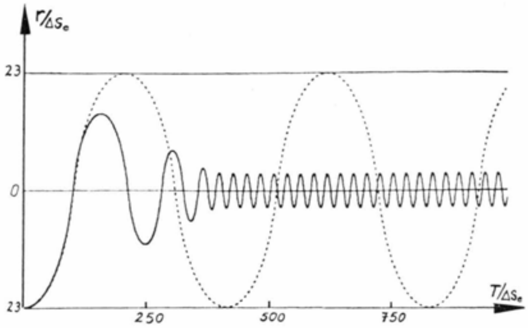
where K must be taken from (8) and the connection between the spatial coordinate r and the auxiliary velocity $w_{(s)}$ is

$$\frac{\mathrm{d}r}{\mathrm{d}s} = \operatorname{Sinh} w_{(s)}\,. \tag{10}$$

The laboratory time $T := c\,t$ is obtainable from

$$\frac{\mathrm{d}T}{\mathrm{d}s} = \operatorname{Cosh} w_{(s)}\,. \tag{11}$$

The subsequent figure exhibits a plot of the solutions for Eqs. (9) to (11), where the horizontal axis shows the lab time in reduced units $(T/\Delta s_{\text{e}})$ and the vertical axis the spatial coordinate r in reduced units $(r/\Delta s_{\text{e}})$. The mass ratio was chosen to be $m_{\text{el}}/m_{\text{mech}} = 0,1$; where $m_{\text{mech}} + m_{\text{el}} = m_{\text{exp}}$ and $m_{\text{exp}}\, c^2\, \Delta s_{\text{e}} = \frac{2}{3} Z^2$ was also used $(\Rightarrow \Delta s = 22\, \Delta s_{\text{e}})$. Moreover, $-Z = +Z^* = |Z|$ = electron charge and $r_0 = \Delta s_{\text{e}}$. We have assumed an initial separation



of $r_{\text{in}} = 23\, \Delta s_{\text{e}}$ between the electron and the potential minimum (at $r = 0$) and that the particle be at rest before it is released at $T = 0$. The dotted curve is the non-radiating limit (6), which clearly yields undamped oscillations. The solid curve is the solution of our problem (9) to (11) and exhibits quite clearly that the radiating particle looses energy by radiation during the first two or three oscillations but is then going over in a stable, radiationless, periodic motion around the potential minimum (straight horizontal line in the middle of the figure).

These quite astonishing results suggest further questions, which are currently under investigation: How many stable states are possible in a given potential well? Are they forming a discrete set? Are there metastable states? Which initial conditions lead to a prescribed final stationary state? Are there also stationary states in more than one dimension?

[1] M. Sorg, Z. Naturforsch. **32 a**, 101 [1977].

[2] J. Petzold, W. Heudorfer, and M. Sorg (im Druck).

[3] M. Sorg, Z. Naturforsch. **31 a**, 664 [1976].

[4] M. Sorg, Z. Naturforsch. **31 a**, 1133 [1976].

[5] P. Caldirola, Nuovo Cim. **3**, Suppl. 2, 297 [1956].